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Identifying an Extra Z^0 Boson with a New Model

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According to the principle of minimality, we find a new $SU(6)$ model. This $SU(6)$ model, and other models, can be identified as a theoretical origin of an extra Z^0 boson. We apply the strategy of Boudjema *et al.* (BLRV) which is very effective in identifying the theoretical origin of an extra Z^0 boson in the new $SU(6)$ model, and compare the model with six other models.

1. INTRODUCTION

An extra weak neutral boson Z^0 was explored by a number of authors over a decade ago (Deshpande and Iskandar, 1979a, b, 1980; Kang and Kim, 1976a, b, 1978; Zee and Kim, 1980; Gao and Wu, 1981). However, at that time there did not exist any experimental information for an extra Z^0. These authors started only from a speculative attitude. If there exist an extra Z^0 boson, it is possible that the Z^0 boson of the standard model (SM) and the extra Z^0 mix. Durkin and Langacker (1986) discussed the neutral current constraint on an extra Z^0 boson and got interesting results. The possibility that the symmetry group is large, such as having an extra $U(1)$ at a scale larger than 200 GeV, is not excluded by available data (Barger *et al.*, 1986). Recent measurements (Amaldi *et al.*, 1987; Costa *et al.*, 1988; Lynn *et al.*, 1988; del Aguila *et al.*, 1991; Aquino *et al.*, 1991; Chiappinelli, 1991; Altarelli *et al.*, 1991) of the properties of the Z^0 boson (at Tevatron, SLC, and LEP colliders) have provided a new means for exploring the numerous extensions of the SM including an extra Z^0. Some authors (Chiappetta *et al.*, 1991; Avera *et al.*, 1991; Frere and Repko, 1991; Gonzalez-Garcia and Valle, 1991) have given a new limit of the mass of

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extra neutral gauge bosons at LHC and SSC using estimated machine luminosities. With the expectation of greatly improved statistics from these colliders in the near future, detailed comparisons between the data and theoretical predictions can be made (Boudjema *et al.*, 1990). In these cases, identifying the theoretical origin of an extra Z^0 boson from the numerous nonstandard models will be necessary and very interesting. The extra Z^0 boson is present in a wide variety of SM extensions, including the left–right symmetric model (LRM) (Mohapatra, 1986), the recently proposed $SU(2)_q \times SU(2)_l \times U(1)_Y$ model (QLM) (Georgi *et al.*, 1989a, b; Bargger and Rizzo, n.d.; Rizzo, 1989); models of the composite nature of the Z^0 (Kuroda *et al.*, 1985; Baur *et al.*, 1987), the model of the strongly-interacting electroweak sector (Casalbuoni *et al.*, 1988), superstring-inspired E_6 models (London and Rosner, 1986), and so on.

Now our question is whether or not there exist yet other models including extra Z^0 bosons (MIEZ) than those mentioned above. In other words, are we missing some MIEZ?

The superstring-inspired E_6 model seemed to be a unique candidate at a certain stage of development of superstring models. In the fast few years the prediction of E_6 arising from superstrings is no longer unique (Bailin *et al.*, 1986a, b; Greene *et al.*, 1986). Superstring models have been proposed that do not require an extra Z^0 (Antoniadis *et al.*, 1987, 1988a, b). Modifications of the conventional supersymmetric picture have also been suggested which require an extra Z^0, but do not have a general E_6 origin (Barbieri and Hall, 1988; Font *et al.*, 1989). It is not yet known how superstrings relate to experiment. I think the superstring-inspired E_6 model ought neither to be unique nor the most promising candidate of MIEZ. My claim is that all MIEZ not excluded by data ought to be candidates. Also, there is no especially interesting motivation for the MIEZ mentioned above.

How can we find those missing models? If a strong basis of the theory and sufficient information of experiment are deficient, then the best method to find those missing models will be to use the principle of minimality. The principle of minimality is a most fundamental principle in the theory of physics, including particle physics. If it were not, the theory of particle physics could not be started from the Lagrangian.

The concrete meanings of the principle of minimality will be as follows:

(i) On one hand, the SM has achieved important success and excellent agreement with all existing experimental data. The data have explicitly shown that there are very important reasonable ingredients in SM. These reasonable ingredients absolutely ought not to be altered in any case. On the other hand, the SM leaves open a number of fundamental problems and contains many undetermined parameters, so we must look for a still more fundamental theory which reduces to the SM at low energies.

How will we look for a more fundamental theory?

One of the methods will be to try a minimal extension of the gauge symmetry on the basis of the SM, which will reduce to the SM at low energies. The SM gauge symmetry is $SU(3)_c \times SU(2)_L \times U(1)_Y$.

(ii) Now we do not know how many extra Z^0 bosons exist. According to the minimal principle, first we ought to research the case that there exists only one extra Z boson. So according to this principle SM ought to be extended to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)$.

(iii) The minimal grand unified theories (GUTs) including $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)$ will be groups of rank five (Amaldi *et al.*, 1987). The $SO(10)$ and $SU(6)$ models are the only GUTs constructed according to general principles (similar to Georgi's principles) (Li, 1988a, b, 1989a, b) that are simple Lie group of rank five. The $SO(10)$ GUTs including LRM have been discussed by many authors. The $SU(6)$ GUTs, not yet ruled out by the experiment, have not been discussed. So, according to the principle of minimality, the $SU(6)$ model is a very natural extension of the SM. The E_6 model will include two extra Z^0 bosons, and so does not agree with the principle of minimality because it will remain one extra Z^0 boson. The E_6 model has more undetermined parameters than $SU(6)$. For example, the mixing angle between the extra Z^0 for E_6 is [we follow the notation of Amaldi *et al.* (1987)] $Z^{0'} = \cos\beta \cdot Z^0_\chi + \sin\beta \cdot Z^0_\psi$.

The extra Z^0 boson of the $SU(6)$ is not the extra Z^0 boson of a specific value of mixing angle β of E_6 because the $SU(6)$ model is not physically interpretable as a subgroup of E_6 because, if it were, there would be two extra Z^0 bosons. Here we will discuss only the case of physics with only one extra Z^0 boson. We will not discuss two extra Z^0 bosons. We do not think that the model with one extra Z^0 boson is a special case of a model with two extra Z^0 bosons. In fact, the E_6 model is not necessary if there is only one extra Z^0 boson in the world.

The GUTs based on $SU(6)$ have already been explored by a number of authors (Baaklini, 1980; Kim and Roiesnel, 1980) in directions different from ours. Most of their models are vectorlike and contain electroweak $SU(3) \times U(1)$. These are now ruled out by neutral current data. The $SU(6)$ model of this paper is different, is not ruled out by the new data, and yields interesting results (Li, 1988a, b, 1989a, b). Its distinguishing features are as follows:

(i) The extra Z^0 boson is larger than 200 GeV. It cannot be included in earlier $SU(6)$ GUTs because the two Z^0 in those theories are broken on the same spontaneous symmetry breaking (SSB) scale.

(ii) This $SU(6)$ model retains all the results of the SM at the SSB scale of $M_{W^\pm, Z_1^0}$ because the SM is in excellent agreement with existing data.

(iii) The extra Z^0 boson and the new fermions appear at the SSB scale of $M_{Z_2^0}(>200\ \mathrm{GeV})$ and the new fermions will not be bizarre with respect to color and flavor of the SM.

(iv) It overcomes the difficulty of the proton decay in $SU(5)$ GUTs.

So it ought to be considered as a candidate MIEZ and identified further as a theoretical origin of an extra Z^0 boson. The motivations of this $SU(6)$ model are of general interest and it has yielded interesting results (Li, 1988a, b, 1989a, b), so it will be worthwhile to make a further analysis.

2. BRIEF INTRODUCTION TO THE PRESENT $SU(6)$ MODEL[2]

In order to get the above physical results for $SU(6)$, the pattern of SSB to be adopted is as follows:

$$\begin{aligned} SU(6) &\xrightarrow{\text{adj.} H_1, M_6} SU(5)\times U(1) \\ &\xrightarrow{\text{adj.} H_2, M_5} SU(3)_c \times \underset{g_2}{SU(2)_L} \times \underset{g_1}{U(1)} \times \underset{g'}{U(1)} \\ &\xrightarrow{\text{vect.} h_1, M_{Z'}} SU(3)_c \times SU(2)_L \times U(1)_Y \\ &\xrightarrow{\text{vect.} h_2, M_Z} SU(3)_c \times U(1)_{em} \end{aligned} \tag{1}$$

These patterns of SSB can easily be realized and we get the masses of the gauge bosons if we use the adjoint Higgs H_i and vector Higgs $h_i (i = 1, 2)$. There are cross coupling between h_1 and h_2. They contain two neutral color triplets of Higgs fields, so their linear combinations ought to be studied. Part of them may be eaten to give mass to the gauge boson. Another part, as physical Higgs fields, may also mediate proton decay, but they must and can be made very massive. The gauge bosons are

$$\mathscr{A} = \frac{1}{\sqrt{2}} \sum_{i=1}^{35} \lambda_i \cdot A_\mu^i \tag{2}$$

[2]For details see Li (1988a, b, 1989a, b).

The diagonal part of the expanded formula of the gauge bosons (2) (to be related to neutral current) may be written

$$\begin{aligned}\text{diag. } \mathscr{A} = \Big(& G_1^1 - \frac{2}{\sqrt{30}} B \mp \frac{1}{10\sqrt{3}} A,\ G_2^2 - \frac{2}{\sqrt{30}} B \mp \frac{1}{10\sqrt{3}} A, \\ & G_3^3 - \frac{2}{\sqrt{30}} B \mp \frac{1}{10\sqrt{3}} A,\ \frac{W^3}{\sqrt{2}} + \frac{3}{\sqrt{30}} B \mp \frac{\sqrt{3}}{5} A, \\ & \frac{-W^3}{\sqrt{2}} + \frac{3}{\sqrt{30}} B \mp \frac{\sqrt{3}}{5} A,\ \frac{\pm\sqrt{3}}{2} A \Big)\end{aligned} \tag{3}$$

The left-handed fermions are assigned to one 15- and both 6*-dimensional representations.
Their explicit forms are

$$(\psi^{ab})_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 & -D^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 & -D^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 & -D^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ & -E^+ \\ d^1 & d^2 & d^3 & e^+ & 0 & E^0 \\ D^1 & D^2 & D^3 & E^+ & -E^0 & 0 \end{pmatrix}_L \tag{4}$$

$$(\psi_{1a})_L = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\gamma_e \\ I^0 \end{pmatrix}_L \tag{5}$$

$$(\psi_{2b})_L = \begin{pmatrix} F^1 \\ F_2 \\ F_3 \\ G^- \\ -G^0 \\ H^0 \end{pmatrix}_L \tag{6}$$

The fermion representation can only form Yukawa coupling with vector Higgs h_i and after SSB we will get the masses of the fermions

$$\begin{aligned} m_d &= m_e = g''u \\ m_u &= 8g''v, \qquad 8v \approx u \end{aligned} \tag{7}$$

The new fermions are broken on the scale $M_{Z_2^0}$, but their masses are all zero because they are all chiral. The q^2 dependence of $\sin^2\theta_w(q^2)$ and $\alpha(q^2)/\alpha_s(q^2)$ can be calculated from the renormalization group equations because that q^2 dependence contains a free parameter M_6^2/M_5^2 which may be accommodated. So the experimental value of the proton decay may be calculated from the theory (Li, 1988a, b, 1989a, b).

3. IDENTIFYING AN EXTRA Z^0 BOSON WITH A NEW MODEL

The strategy of Boudjema *et al.* (1990) (BLRV) is very effective for identifying a theoretical origin of an extra Z^0 boson in a wide variety of models. The BLRV strategy is expressed by curves (or strips) of the $R_{5,6}$ versus $\Gamma_{Z_2^0\to\mu\bar{\mu}}/M_{Z_2^0}$ plane, where $\Gamma_{Z_2^0\to\mu\bar{\mu}}$ is the partial width of the Z_2^0 decay into the muonic pair,

$$\Gamma_{Z_2^0\to\Sigma_{i=1}^{5,6} q_i\bar{q}_i}$$

is the partial width of the Z_2^0 decay into the five known or six quarks pair, and

$$R_{5,6}\equiv\Gamma_{Z_2^0\to\Sigma_{i=1}^{5,6} q_i\bar{q}_i}/\Gamma_{Z_2^0\to\mu\bar{\mu}}$$

This strategy requires the preliminary measurement of the muonic pair width $\Gamma_{Z_2^0\to\mu\bar{\mu}}$ and of the ratio $R_{5,6}$ of the Z_2^0 resonance. They worked in the Born approximation, neglecting one-loop radiative corrections, whose the effect is smaller than the experimental errors of the various widths and ratios. In the $(R_{5,6}, \Gamma_{Z_2^0\to\mu\bar{\mu}}/M_{Z_2^0})$ plane the two extra gauge models and the four alternative models belong to completely different regions except for their Z_V^0 boson. In order to differentiate among the three candidate models they further discussed longitudinal polarized asymmetries. The direct production of an extra Z boson will be problematic both for future $p\bar{p}$ colliders and LEP for $M_{Z_2^0}\geqslant 200$ GeV. If the extra Z^0 is in the considered mass range, 400 GeV $\leqslant m_{Z_2}\leqslant$ 1 TeV, then it will be possible to discover an extra Z^0 boson with a future e^+e^- collider with total energy up to 1 TeV and the measurement of its partial width including the top quark will also be possible.

The BLRV strategy has been applied to six MIEZ and has very well differentiated these models, so it will be very clean and convenient to make detailed comparisons between the data and theoretical predictions. However, the $SU(6)$ model as a candidate of MIEZ has not yet been identified using their strategy. It is worthwhile to make the analysis before mentioned, otherwise we will miss one candidate. Let us analyze the $SU(6)$ model.

Let us first quickly list the theoretical expressions of the relevant quantities.

In the tree approximation, the Z_2^0 (extra Z^0) boson coupling to charged fermions in the $SU(6)$ model is defined as

$$\mathscr{L}_{Z_2^0 \to f\bar{f}} = G_2 J^{\mu}_{Z_2^0} Z^0_{2\mu} \tag{8}$$

$$G_2 J^{\mu}_{Z_2^0} = \sum_f \bar{f}\gamma^{\mu}[v_f(Z_2^0) + \gamma^5 a_f(Z_2^0)] f \tag{9}$$

$$v_\nu(Z_2^0) = -a_\nu(Z_2^0) = \frac{-1}{4}(g_1^2 + g_2^2)^{1/2} \sin\varphi \tag{10}$$

$$v_l(Z_2^0) = \frac{1}{4}(g_1^2 + g_2^2)^{1/2} \sin\varphi(1 - 4\sin^2\theta_W) \tag{11}$$

$$a_l(Z_2^0) = \frac{-1}{4}(g_1^2 + g_2^2)^{1/2} \sin\varphi \tag{12}$$

$$v_u(Z_2^0) = \frac{1}{4}\left[\mp\cos\varphi \cdot g' + (g_1^2 + g_2^2)^{1/2} \sin\varphi\left(\frac{8}{3}\sin^2\theta_W - 1\right)\right] \tag{13}$$

$$a_u(Z_2^0) = \frac{1}{4}[\mp 3g'\cos\varphi + (g_1^2 + g_2^2)^{1/2} \sin\varphi] \tag{14}$$

$$v_d(Z_2^0) = \frac{1}{4}\left[\pm 2g'\cos\varphi + (g_1^2 + g_2^2)^{1/2} \sin\varphi\left(1 - \frac{4}{3}\sin^2\theta_W\right)\right] \tag{15}$$

$$a_d(Z_2^0) = \frac{-1}{4}(g_1^2 + g_2^2)^{1/2} \sin\varphi \tag{16}$$

$$v_D(Z_2^0) = -a_D(Z_2^0) = \frac{1}{4}\left[\mp g'\cos\varphi - \frac{2}{3}(g_1^2 + g_2^2)^{1/2} \sin\varphi \sin^2\theta_W\right] \tag{17}$$

$$v_{E^+}(Z_2^0) = -a_{E^+}(Z_2^0) = \frac{1}{4}[\mp 3g'\cos\varphi + (g_1^2 + g_2^2)^{1/2} \sin\varphi(2\sin^2\theta_W - 1)] \tag{18}$$

$$v_{E^0}(Z_2^0) = -a_{E^0}(Z_2^0) = \frac{1}{4}[\mp 3g'\cos\varphi + (g_1^2 + g_2^2)^{1/2} \sin\varphi] \tag{19}$$

$$v_F(Z_2^0) = -a_F(Z_2^0) = \frac{1}{4}[\mp g'\cos\varphi + (g_1^2 + g_2^2)^{1/2} \sin\varphi \sin^2\theta_W] \tag{20}$$

$$v_{f^0}(Z_2^0) = -a_{f^0}(Z_2^0) = \pm\frac{3}{4}g'\cos\varphi \tag{21}$$

$$v_{H^0}(Z_2^0) = -a_{H^0}(Z_2^0) = \pm\frac{3}{4}\, g' \cos\varphi \tag{22}$$

$$v_{G^0}(Z_2^0) = -a_{G^0}(Z_2^0) = \frac{-1}{4}\,(g_1^2 + g_2^2)^{1/2} \sin\varphi \tag{23}$$

$$v_{G^-}(Z_2^0) = -a_{G^-}(Z_2^0) = \frac{1}{4}\,(g_1^2 + g_2^2)^{1/2} \sin\varphi(1 - 2\sin^2\theta_W) \tag{24}$$

In equations (10)–(24), g_2, g_1, and g' are the coupling constants that couple the gauge bosons of the corresponding gauge group to fermions; $\sin^2\theta_W$ is the Weinberg angle; φ is the mixing angle of the Z^0 and $Z^{0'}$,

$$\begin{aligned} Z_1^0 &= \cos\varphi \cdot Z^0 + \sin\varphi \cdot Z^{0'} \\ Z_2^0 &= \sin\varphi \cdot Z^0 + \cos\varphi \cdot Z^{0'} \end{aligned} \tag{25}$$

In (25), Z^0 is the Z^0 boson of the SM; $Z^{0'}$ is the extra Z^0 boson that does not mix with Z^0.

The expression of the Z_2^0 width on the resonance is

$$\begin{aligned} \Gamma_{Z_2^0 \to f\bar{f}} = \frac{m_{Z_2^0}}{12\pi}\left(1 - \frac{4m_f^2}{m_{Z_2^0}^2}\right)^{1/2} \Big\{ &([v_f(Z_2^0)|^2 + |a_f(Z_2^0)|^2] \\ &+ \frac{2m_f^2}{m_{Z_2^0}^2}[|v_f(Z_2^0)|^2 - 2\,|a_f(Z_2^0)|^2]\Big\} \end{aligned} \tag{26}$$

If $m_{Z_2^0}^2 > 2m_t^2$, that is, the top mass is sufficiently smaller than the Z_2^0 mass, then equation (26) may be reduced to

$$\Gamma_{Z_2^0 \to f\bar{f}} = \frac{m_{Z_2^0}}{12\pi}\,[(v_f(Z_2^0)|^2 + |a_f(Z_2^0)|^2] \tag{27}$$

We can derive the following relevant formulas from equation (27):

$$\frac{\Gamma_{Z_2^0 \to \mu\bar{\mu}}}{m_{Z_2^0}} = \frac{(g_1^2 + g_2^2)(1 - 2\sin^2\theta_W)}{96\pi}\sin^2\varphi \tag{28}$$

$$\begin{aligned} R_5 = \frac{1}{2(1 - 2\sin^2\theta_W)}\Bigg\{ &\frac{g'^2(1 - 2\sin^2\theta_W)}{3\pi}\,\frac{1}{(\Gamma_{\mu\bar{\mu}}/m_{Z_2^0})} \\ &\mp \frac{4g'(5 + 4/3\sin^2\theta_W)}{(g_1^2 + g_2^2)^{1/2}}\left[\frac{(g_1^2 + g_2^2)(1 - 2\sin^2\theta_W)}{96\pi(\Gamma_{\mu\bar{\mu}}/m_{Z_2^0})} - 1\right]^{1/2} \\ &- \frac{32g'}{g_1^2 + g_2^2} + 2\left(\frac{8}{3}\sin^2\theta_W - 1\right)^2 + 3\left(\frac{4}{3}\sin^2\theta_W - 1\right)^2 + 5\Bigg\} \end{aligned} \tag{29}$$

$$R_6 = \frac{1}{2(1-\sin^2\theta_W)}\left\{\frac{7g'^2(1-\sin^2\theta_W)}{48\pi(\Gamma_{\mu\bar{\mu}}/m_{Z_2^0})} \mp \frac{8g'}{(g_1^2+g_2^2)^{1/2}}\right.$$
$$\times\left[\frac{(g_1^2+g_2^2)(1-2\sin^2\theta_W)}{96\pi(\Gamma_{\mu\bar{\mu}}/m_{Z_2^0})} - 1\right]^{1/2}$$
$$\left. - \frac{14g'^2}{g_1^2+g_2^2} + \left(\frac{8}{3}\sin^2\theta_W - 1\right)^2 + \left(\frac{4}{3}\sin^2\theta_W - 1\right)^2 + 2\right\} \qquad (30)$$

In equations (29)–(30), we have used $\sin^2\theta_W = 0.230$; the $g_2(q^2)$, $g_1(q^2)$, and $g'(q^2)$ are determined by the equations of the renormalization group using the experimental values as input. These are shown in Figs. 1 and 2. The origin of the difference in curves 1–4 in Figs. 1 and 2 arises in the signs ($\mp$) before the A neutral boson in equation (3). The selection of the sign will be determined by the experimental point that belongs to one of the four characteristic curves. If the experimental point indicates a Z_2^0 of $SU(6)$ origin, then equation (28) can be used to determine the value of the mixing angle $\sin^2\varphi$ of the Z^0 and $Z^{0'}$. Equations (29) and (30) do not include any free parameters. They differ from equations (25), (26), (28), and (29) of Boudjema *et al.* (1990) in that the latter include the mixing angle θ_M (in

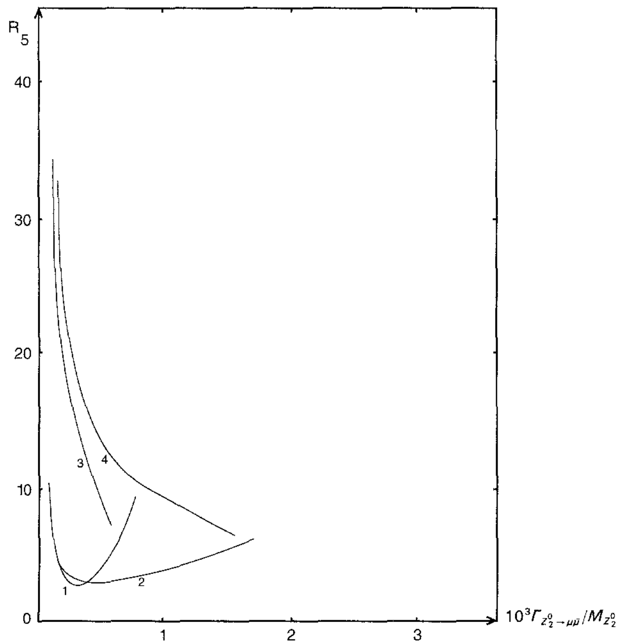

Fig. 1. The ratio R_5 versus $\Gamma_{Z_2^0\to\mu\bar{\mu}}/M_{Z_2^0}$ for the $SU(6)$ model.

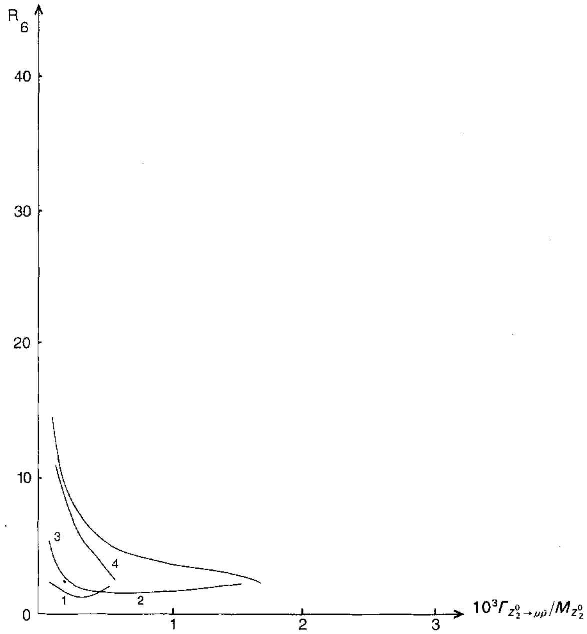

Fig. 2. The ratio R_6 versus $\Gamma_{Z_2^0 \to \mu\bar{\mu}}/M_{Z_2^0}$ for the $SU(6)$ model.

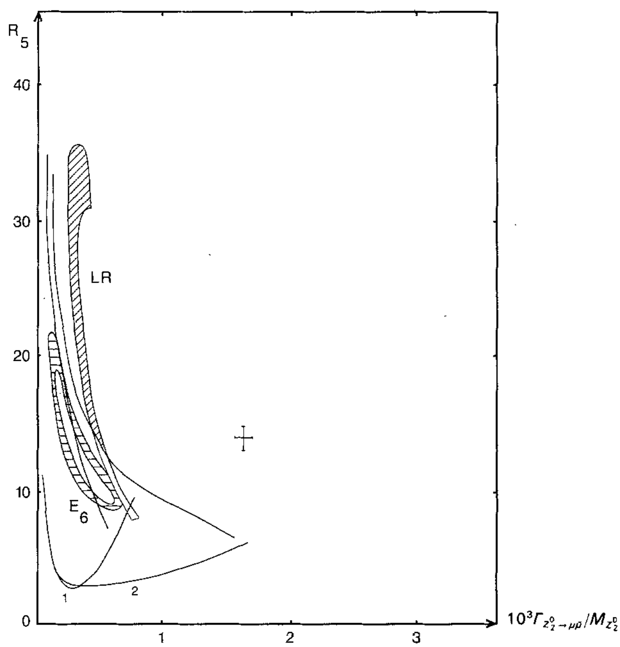

Fig. 3. The ratio R_5 versus $\Gamma_{Z_2^0 \to \mu\bar{\mu}}/M_{Z_2^0}$ for the $SU(6)$, E_6, and LR models.

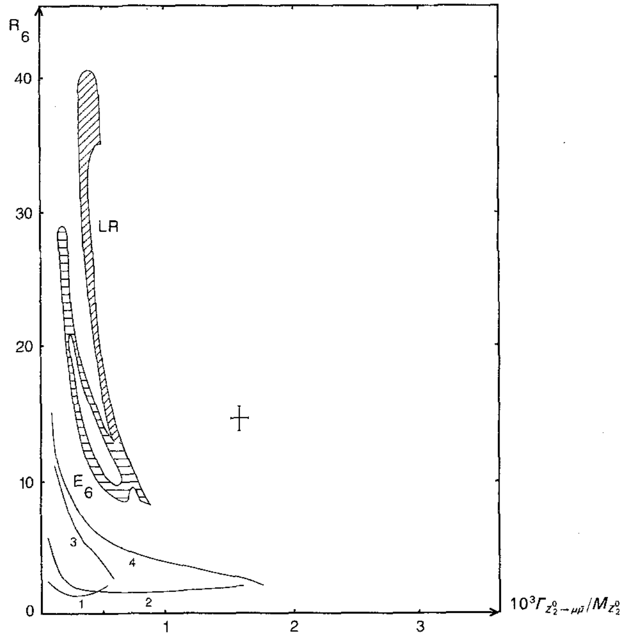

Fig. 4. The ratio R_6 versus $\Gamma_{Z_2^0 \to \mu\bar{\mu}}/M_{Z_2^0}$ for the $SU(6)$, E_6, and LR models.

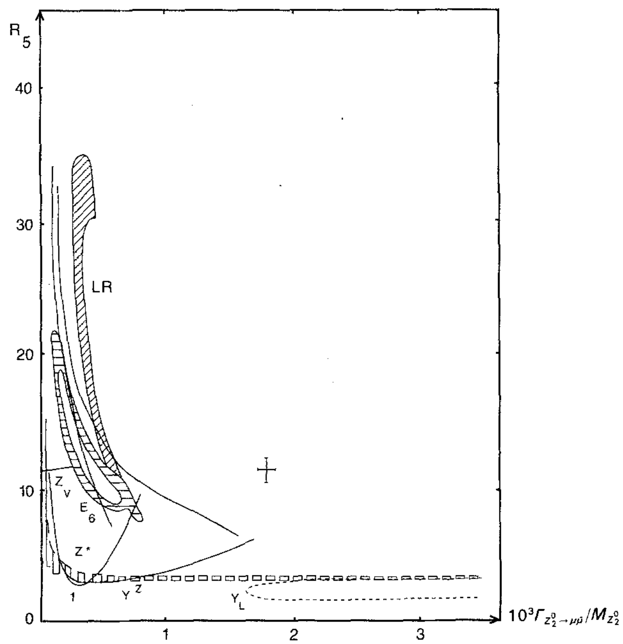

Fig. 5. The ratio R_5 versus $\Gamma_{Z_2^0 \to \mu\bar{\mu}}/M_{Z_2^0}$ for the $SU(6)$, E_6, LRM, and other four models.

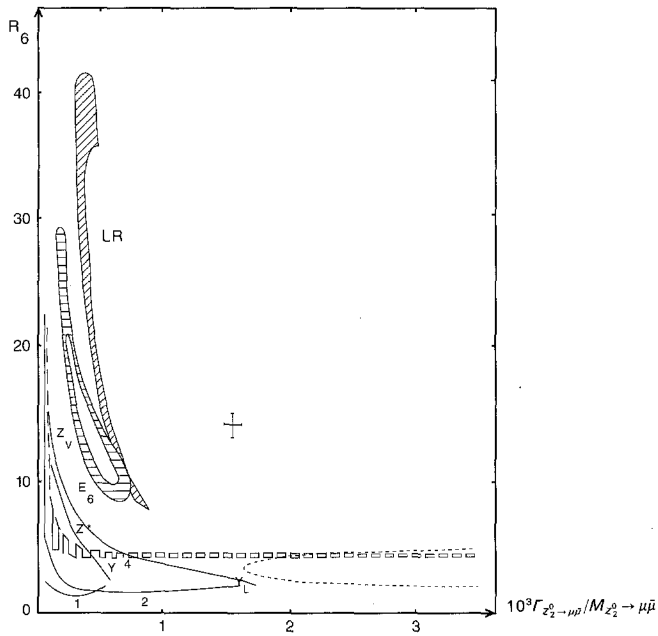

Fig. 6. The ratio R_6 versus $\Gamma_{Z_2^0\to\mu\bar{\mu}}/M_{Z_2^0}$ for the $SU(6)$, E_6, LRM, and other four models.

zeroth order) and β. So the curves in Figs. 1 and 2 cannot be transformed into strips. Figure 3 (Fig. 4) is a comparison between Fig. 1 (Fig. 2) and Fig. 4 (Fig. 5) of Boudjema *et al.* (1990). Curves 1 and 4 in Fig. 1 and the strip of LRM in Fig. 4 of Boudjema *et al.* (1990) have one common intersection. Curve 3 in Fig. 1 and the strip of the E_6 in Fig. 4 of Boudjema *et al.* (1990) have a common intersection. The curves in Fig. 2 and the strips in Fig. 5 of Boudjema *et al.* (1990) do not any common intersection. Figure 5 (Fig. 6) is a comparison between Fig. 1 (Fig. 2) and Fig. 6 (Fig. 7) of Boudjema *et al.* (1990). Because three composite models (Y, Y_L, Z^*) have not yet been differentiated in Figs. 5 and 6, curves 1–4 with their common intersection are confused. BLRV used a polarized asymmetric method to eliminate the confusion of the composite models [Fig. 8 and 9 in Boudjema *et al.* (1990)], so we will use the polarized asymmetric method for this $SU(6)$ model in order to get rid of the confusion of the three composite models with curves 1–4. Based on the general method (Boudjema *et al.*, 1990) of polarized asymmetries on Z_2^0 and equations (11)–(16), it will be very easy to get formulas for the three polarized asymmetries for this $SU(6)$ model. We have

$$A_{LR}^{\prime h(SU_6)} \equiv A_e^{\prime (SU_6)} = \frac{N_L - N_R}{N_L + N_R} \simeq \frac{2v_l a_l}{v_l^2 + a_l^2} = \frac{-2(1-4\sin^2\theta_W)}{(1-4\sin^2\theta_W)^2+1} \tag{31}$$

$$A_{FB}^{\prime u(SU_6)} \equiv A_u^{\prime (SU_6)} = \frac{3}{2}\frac{v_u a_u}{(v_u^2 + a_u^2)}$$

$$= \frac{3}{2}\frac{\begin{pmatrix}[\mp g(\cos\varphi)/(1-\cos^2\varphi)^{1/2} + 8/3\sin^2\theta_W - 1] \\ \times[\mp 3g(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+1]\end{pmatrix}}{\begin{pmatrix}[\mp g(\cos\varphi)/(1-\cos^2\varphi)^{1/2} + 8/3\sin^2\theta_W - 1]^2 \\ +[\mp g(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+1]^2\end{pmatrix}} \tag{32}$$

$$A_{FB}^{\prime d(SU_6)} \equiv A_d^{\prime (SU_6)} = \frac{3}{2}\frac{v_d a_d}{(v_d^2 + a_d^2)}$$

$$= -\frac{3}{2}\frac{\pm 2g(\cos\varphi)/(1-\cos^2\varphi)^{1/2} + 1 - 4/3\sin^2\theta_W}{[\pm 2g(\cos\varphi)/(1-\cos^2\varphi)^{1/2} + 1 - 4/3\sin^2\theta_W] + 1} \tag{33}$$

where

$$g \equiv g'/(g_1^2 + g_2^2)^{1/2}$$

If we substritute 0.23 for $\sin^2\theta_W$, then equations (31)–(33) become

$$A_e^{\prime (SU_6)} \simeq -0.6 \tag{34}$$

$$A_u^{\prime (SU_6)} \simeq \frac{3}{2}\frac{\begin{pmatrix}[\mp g(\cos\varphi)/(1-\cos^2\varphi)^{1/2} - 0.387] \\ \times[\mp 3g(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+1]\end{pmatrix}}{\begin{pmatrix}[\mp g(\cos\varphi)/(1-\cos^2\varphi)^{1/2} - 0.387]^2 \\ +[\mp 3g(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+1]^2\end{pmatrix}} \tag{35}$$

$$A_d^{\prime (SU_6)} \simeq \frac{-3}{2}\frac{\pm 3g(\cos\varphi)/(1-\cos^2\varphi)^{1/2} + 0.693}{[\pm 2g(\cos\varphi)/(1-\cos^2\varphi)^{1/2} + 0.693]^2 + 1} \tag{36}$$

where g is a running coupling constant. If $\alpha_3((34\,\text{GeV})^2) = 0.136$, $\alpha^{-1}(m_W^2) = 128$, and $M_6/M_5 = 1.23$ are inputs for the renormalization group equations for the pattern of SSB of equation (1), then we get

$$g = 0.32 \tag{37}$$

or

$$g = 0.188 \tag{38}$$

where the two values of g come from different signs before A of equation (3). If we put $g = 0.32$ in equations (35) and (36), then we get

$$A_u^{\prime(SU_6)} \simeq \frac{3}{2} \frac{\left(\begin{array}{l}[\mp 0.32(\cos\varphi)/(1-\cos^2\varphi)^{1/2}-0.387] \\ \quad \times[\mp 0.96(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+1]\end{array}\right)}{\left(\begin{array}{l}[\mp 0.32(\cos\varphi)/(1-\cos^2\varphi)^{1/2}-0.387]^2 \\ \quad +[\mp 0.96(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+1]^2\end{array}\right)} \tag{35'}$$

$$A_d^{\prime(SU_6)} \simeq \frac{-3}{2} \frac{[\pm 0.64(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+0.693]}{[\pm 0.64(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+0.693]^2+1} \tag{36'}$$

If we put $g=0.188$ in equations (35) and (36), then we get

$$A_u^{\prime(SU_6)} \simeq \frac{3}{2} \frac{\left(\begin{array}{l}[\mp 0.188(\cos\varphi)/(1-\cos^2\varphi)^{1/2}-0.387] \\ \quad \times[\mp 0.564(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+1]\end{array}\right)}{\left(\begin{array}{l}[\mp 0.188(\cos\varphi)/(1-\cos^2\varphi)^{1/2}-0.387]^2 \\ \quad +[\mp 0.564(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+1]^2\end{array}\right)} \tag{35''}$$

$$A_d^{\prime(SU_6)} \simeq \frac{-3}{2} \frac{\pm 0.376(\cos\varphi)/(1-\cos^2\varphi)^{1/2}+0.693}{[\pm 0.376(\cos\varphi)^{1/2}/(1-\cos^2\varphi)^{1/2}+0.693]^2+1} \tag{36''}$$

It is obvious that $A_e^{\prime(SU_6)}$ is a constant, and that $A_{u,d}^{\prime(SU_6)}$ are functions only of $\cos\varphi$, the mixing angle of Z^0 and $Z^{0'}$. If the value of φ has been determined, then $A_{u,d}^{\prime(SU_6)}$ are also constant, so they are only a point on the $A_{u,d}^{\prime(SU_6)}$ versus $A_e^{\prime(SU_6)}$ plane. If the value of φ is limited to a small range, then it will be a small segment of the straight line on the $A_{u,d}^{\prime(SU_6)}$ versus $A_e^{\prime(SU_6)}$ plane. However, when the value of φ is not known, its maximum range will be a straight line on the $A_{u,d}^{\prime(SU_6)}$ versus $A_e^{\prime(SU_6)}$ plane. So it is different from Figs. 8 and 9 of BLRV, and the $A_{u,d}^{\prime(SU_6)}$ are not functions of

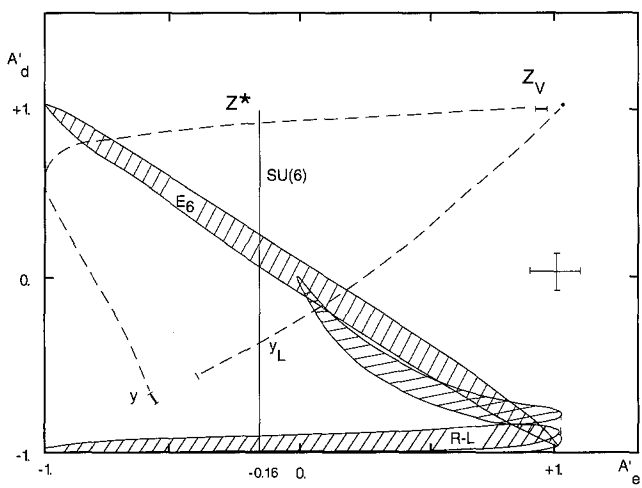

Fig. 7. A'_d versus A'_e for the seven candidate models.

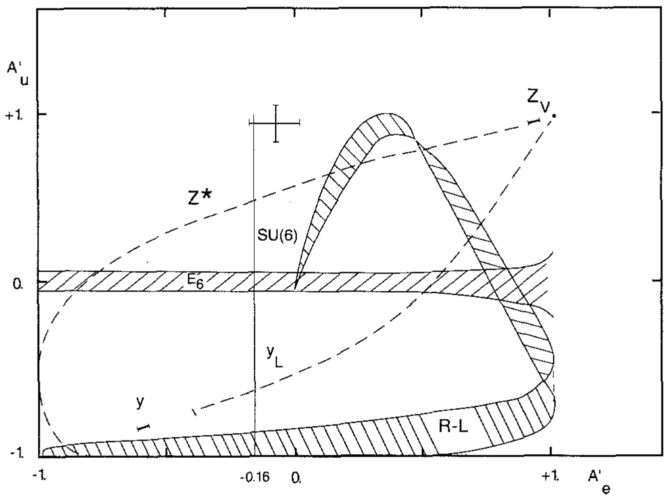

Fig. 8. A'_u versus A'_e for the seven candidate models.

the $A_e'^{(SU_6)}$. A comparison on the $A'_{u,d}$ versus A'_e plane between the $SU(6)$ model and the six models of BLRV is given in Figs. 7 and 8. It appears at a glance that the straight line of $A_{u,d}'^{(SU_6)}$ versus $A_e'^{(SU_6)}$ has four common intersections corresponding to four models (LRM, Y_L, E_6, $Z^{0}*$). However, the four common intersections are not certainly a real point of $A_{u,d}'^{(SU_6)}$ versus $A_e'^{(SU_6)}$, because the value of φ is not yet known. Those real points will remain on the straight line of $A_{u,d}'^{(SU_6)}$ versus $A_e'^{(SU_6)}$.

4. SUMMARY

1. According to the principle of minimality, we find a missing $SU(6)$ model including an extra Z^0 boson. This $SU(6)$ model as well as other candidate models can be identified as a theoretical origin of an extra Z^0 boson.

2. The $SU(6)$ model yields interesting results. Its motivation is in general interesting. It possesses distinguishing features and is not ruled out by data.

3. Because the strategy of BLRV is very effective in identifying the theoretical origin of an extra Z^0 boson, and because this $SU(6)$ model ought to be a candidate, we apply the BLRV strategy to this $SU(6)$ model. Finally, we compare the results of this $SU(6)$ model with the results of the other six models.

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